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Comparison of aberration between axicon and lens

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Abstract

We compared aberrations produced by an axicon and a lens. Using the wave aberration theory and the point spread function of a coherent system, we analyzed and compared the light intensity distributions of the spot created by an axicon and by a lens aberrated by astigmatism and coma. We also experimentally observed the aberrated light spots created by an axicon. We found that the beam spot of an axicon is affected only by astigmatism and not by coma, while the point created by a lens is blurred by both astigmatism and coma. This implies that the scanning optical system that incorporates an axicon is easy to correct the aberrations and achieves high resolution and high contrast imaging with a wide field of view. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

In 1987, Durnin reported the theoretical existence of nondiffracting waves [1]. He also demonstrated experimentally the existence of a J_0-beam, which is the simplest form of nondiffracting wave, using an annular aperture and a Fourier-transform lens [2]. To produce nondiffracting waves, many methods have been proposed, including those employing holograms [3], spherical aberration lenses [4], laser cavities [5], spatial light modulators [6], and axicons.

The axicon, which is a kind of conical prism, was first introduced by McLeod in 1954 [7]. The J_0-beam is produced behind the axicon by illuminating the axicon with a collimated laser beam. The use of an axicon provides the most efficient (throughput-advantageous) method for producing the J_0-beam, because the axicon has a higher transmittance than an annular aperture and produces no higher-order diffracted lights as are produced in systems such as those employing a holographic axicon or axicon gratings.

Arimoto proposed utilizing an annular aperture in the laser scanning microscope for increasing the depth of focus [8]. The advantage of using an annular pupil in the excitation optics lies in its long depth of focus as well as its high lateral resolution. However, the annular aperture stops a large amount of the incident laser beam, which is an essential disadvantage of the system involving the annular aperture. Accordingly, in our previous paper, we proposed the use of axicon instead of an

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annular pupil for a scanning optical system and reported the imaging properties of such a system [9]. In the laser scanning microscope, the laser beam is directed onto the objective lens obliquely by a beam scanner for scanning the laser beam spot on the sample plane. It is, therefore, important to study the properties of the aberration produced by an obliquely illuminated axicon. Theoretical and experimental analyses of the diffraction properties of an axicon under the condition of oblique illumination have been reported by Fujiwara [10] and Zhao and Li [11]. However, the distorted patterns of light created by the axicon have not been discussed in terms of these aberrations. In this work, we utilized the wave aberration theory and the point spread function (PSF) of the coherent system to study the aberration of the axicon, and compared the aberrations produced by the axicon with those created by a lens. A comparison of the aberrations is also made experimentally and the results are compared with the results of theoretical analysis.

2. Theoretical analysis

Fig. 1 shows the coordinates of a coherent diffraction-limited system. The axicon or lens is illuminated by a monochromatic plane wave. It is well known that, the light distribution in the focal plane of a lens that is illuminated by a collimated light beam is identical to the Fraunhofer diffraction pattern of a circular aperture, which is known as the Airy pattern. Therefore, as shown in Fig. 1(b), the lens is equivalent to a circular pupil and a Fourier-transform lens. On the other hand, as shown in Fig. 1(c), the axicon is equivalent to the annular aperture with a Fourier-transform lens [2].

The intensity point spread function (PSF) of the coherent system is given by

$$\mathrm{PSF}(x,y) = |P(x,y)|^2, \tag{1}$$

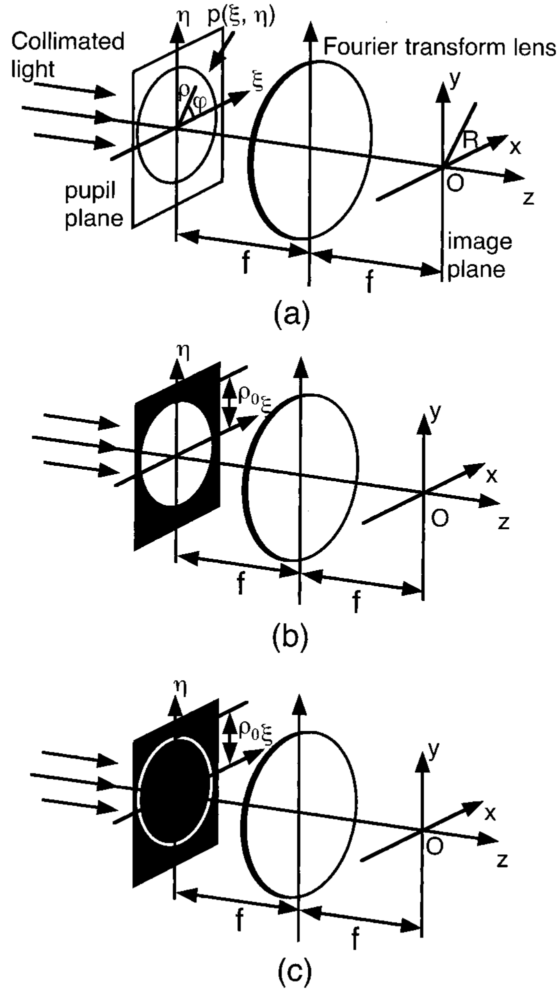

Fig. 1. Notation used to calculate the intensity distribution produced by an axicon and a lens, both illuminated by a monochromatic plane wave. (b) is for an axicon and (c) is for a lens.

where the amplitude PSF, $P(x,y)$, is the Fourier transform of the pupil function $p(\xi,\eta)$:

$$P(x,y) = \int\int_{-\infty}^{\infty} p(\xi,\eta)\mathrm{e}^{-\mathrm{j}(\xi x+\eta y)}\,\mathrm{d}\xi\,\mathrm{d}\eta. \tag{2}$$

Taking the aberrations into account, the pupil function $p(\xi,\eta)$ is written as

$$p(\xi,\eta) = \begin{cases} |p(\xi,\eta)|\exp\{\mathrm{i}\frac{2\pi}{\lambda}W(\rho,\varphi:R)\} & \text{at points in the opening,} \\ 0 & \text{at points outside opening,} \end{cases} \tag{3}$$

where $W(\rho, \varphi : R)$ is a wave aberration [12]. The primary wave aberrations are written as

$$W(\rho, \varphi : R) = b_1\rho^2 + b_2R\rho\cos\varphi + c_1\rho^4 + c_2R^2\rho^2\cos^2\varphi + c_3R^2\rho^2 + c_4R^3\rho\cos\varphi + c_5R\rho^3\cos\varphi,$$
$$\rho = \sqrt{\xi^2 + \eta^2},$$
$$\varphi = \tan^{-1}\frac{\eta}{\xi}, \quad (4)$$

where R is the distance of the spots from the origin of the image plane. The third, fourth, fifth, sixth, and seventh terms of Eq. (4) are the Seidel aberrations, and each indicating spherical aberration, astigmatism, curvature of field, distortion, and coma, respectively, and the first and second terms represent defocus and image shift (wavefront tilt), respectively [13].

3. Computer simulation

We calculated the effects of the astigmatism ($\rho^2\cos^2\varphi$) and the coma ($\rho^3\cos\varphi$) on the focused spots created by the axicon. Using Eqs. (1)–(4), we obtained the intensity distributions of the axicon's points of illumination numerically. Fig. 2 is a pupil function of the axicon with astigmatism. As mentioned above, the axicon is equivalent to the infinitely small annular width using the Fourier transform lens, hence the pupil function of the axicon has value only inside the opening of the annulus, whose radius is ρ_0. Fig. 2(a) and (b) represents amplitude and phase, respectively. The amplitude of the pupil function inside the opening is unity and outside the opening is 0. The phase term of the function is seen to curve according to Eq. (4). Fig. 3 shows the numerical results of the PSF. The intensities of each photograph are saturated to show the side-lobes. Fig. 3(a) shows a beam spot without any aberrations. The intensity distribution of the spot is circularly symmetric and equal to the zeroth-order Bessel function. The astigmatism increases as we go from (b) to (f). The multiplied factors (c_2R^2) of Fig. 3(a)–(f) are 0.0, 7.0, 14.0, 21.0, 31.0, and 55.0, respectively. It is seen that when only a small amount of the astigmatism is present, the beam spot maintains a concentric central point (Fig. 3(b)). However, as the astigmatism increases, the spots are increasingly blurred symmetrically in both the x and y directions and the concentric central rings are broken into many points.

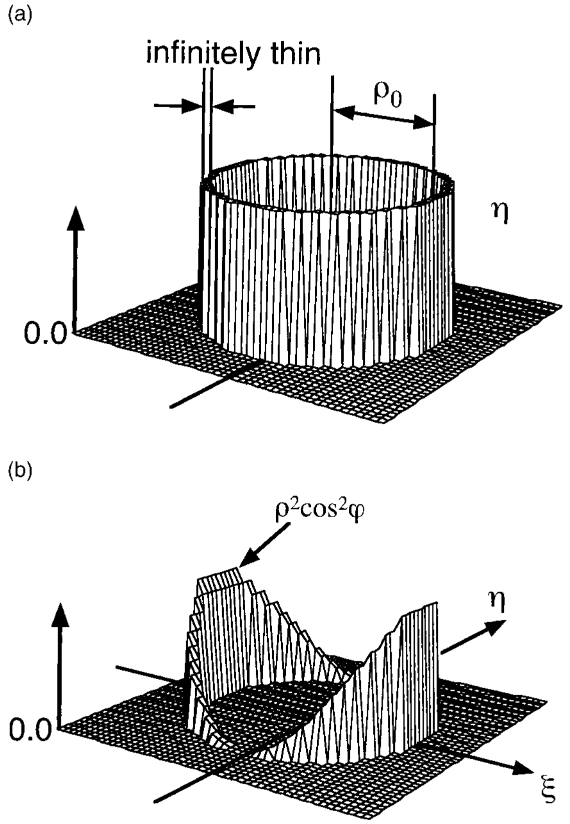

Fig. 2. Pupil function of the axicon with astigmatism: (a) amplitude and (b) phase.

Comparing the astigmatism of the axicon with that of the lens, we found that the aberrated spots of the axicon spread symmetrically in both the x and y directions, while the typical astigmatism spot of a lens in a plane containing a focal line is lengthened primarily in the x-direction with the astigmatism [14].

Fig. 4 shows the pupil function of the axicon with the coma. The η-axis is the axis of rotation of the axicon. Fig. 5 is the numerical results of the aberrated pattern of the axicon. Because the coma shifts the spots' position as well as their

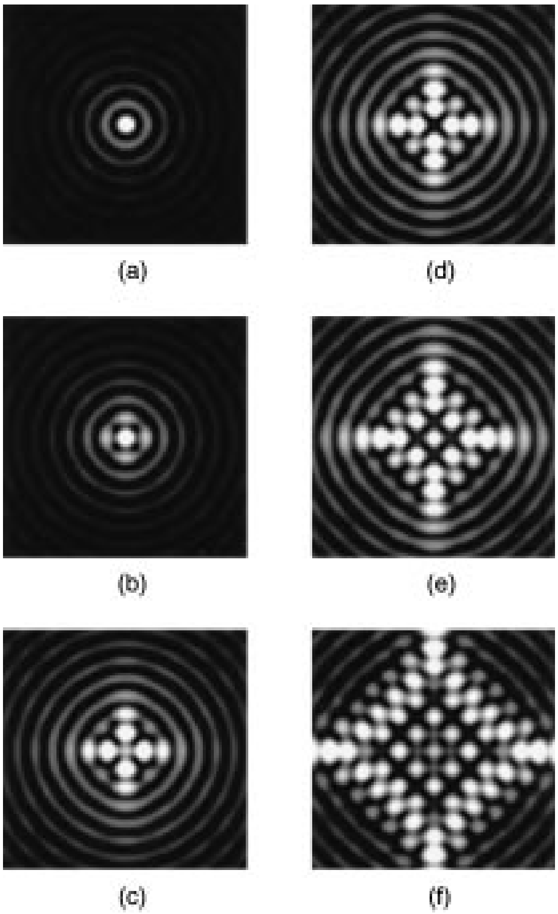

Fig. 3. Numerical simulation of the diffraction pattern of the axicon with astigmatism: (a) astigmatism factor $c_2R^2 = 0.0$, (b) $c_2R^2 = 7.0$, (c) $c_2R^2 = 14.0$, (d) $c_2R^2 = 21.0$, (e) $c_2R^2 = 31.0$, and (f) $c_2R^2 = 55.0$. (The peak of the spot on the photograph is saturated to make the outer ring appear.)

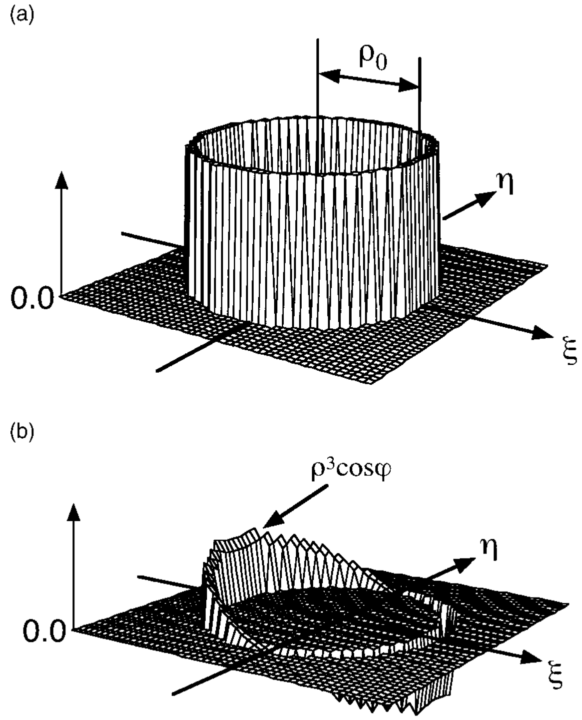

Fig. 4. Pupil function of the axicon with coma: (a) amplitude and (b) phase.

distortion, each spot in Fig. 5 is centered in the photographs. In Fig. 5, we found that even if the coma aberration is applied to the pupil function of the axicon, its spots remain concentric; this means that the coma aberration has no effect on the intensity distribution of the spots created by the axicon. On the other side, the patterns created by the lens are blurred asymmetrically like a comet [14]. This result can be explained as follows: with respect to the axicon's spot, the coma aberration causes only the pattern shift and has no effect on the pattern shape, because the radius ρ of the pupil function is held constant at ρ_0 and the aberration $\rho_0^3 \cos\varphi$ behaves only as the tilt term of the wavefront. On the other hand, for the lens point, since each beam of light transmitted through rings of different radius ρ falls at a different position, if ρ takes on all possible values from 0 to ρ_0, then each spot produced from a different ρ radius falls at a different position and they are added coherently. Hence, the spot is blurred and becomes comet-like.

From these results, we arrived at the conclusion that the axicon does not have odd-order aberrations, which are represented by a coma, and its spot is affected only by even-order aberrations.

4. Experimental results

We observed the aberrated spots of the axicon under experimental conditions. Fig. 6(a) shows the experimental setup for observing the intensity distribution of the aberrated patterns created by the axicon. The axicon was illuminated with a collimated He–Ne laser ($\lambda = 632.8$ nm). We

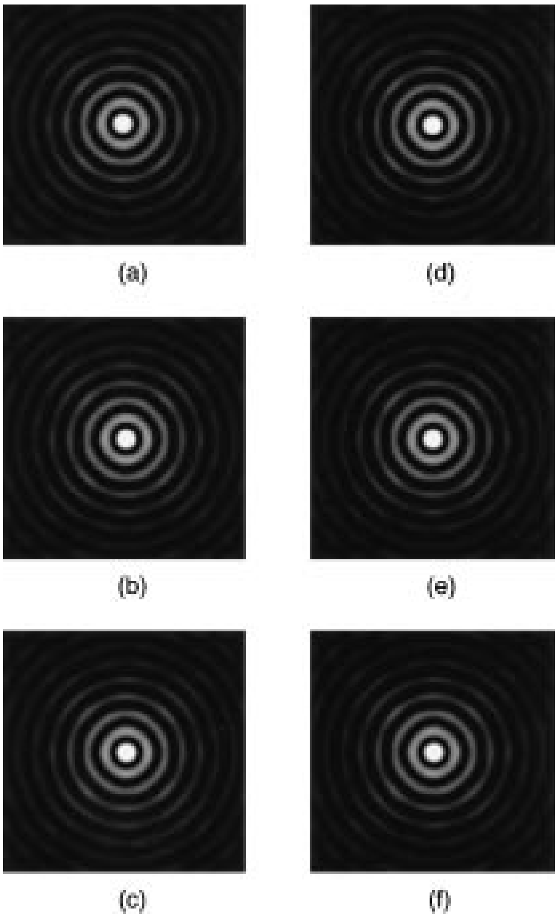

Fig. 5. Numerical results of the central light point of the axicon with coma.

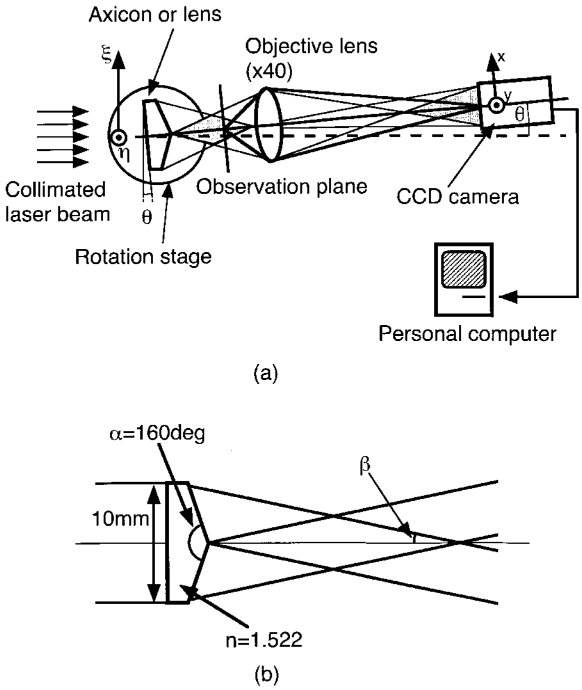

Fig. 6. Optical setup used to observe an aberrated light point produced by a tilted axicon.

obtained the intensity distribution of the focused spots using a microscope (40×magnification) and a CCD camera with its observation plane adjusted to $z = 30$ mm from the apex of the axicon. We used an axicon with parameters of $n = 1.522$ (BK7), $\alpha = 160°$, and a diameter of 10 mm as shown in Fig. 6(b). The angle β is 5.3°, corresponding to a numerical aperture of 0.09. The axicon is tilted by a rotation stage. Fig. 7 shows the experimental results created by oblique plane-wave illumination. The tilting angles of the axicon in Fig. 7(a)–(f) are $\theta = 0.0°$, 5.0°, 6.0°, 7.0°, 8.0°, and 10.0°, respectively. The intensity of each photograph is normalized by its maximum light intensity to enhance the image of the outer rings. When $\theta = 0.0°$, the patterns are circularly symmetric and their intensity distribution is equal to the zeroth-order Bessel function. The intensity distributions of the axicon spots increasingly display aberration, and with increasing axicon rotation, we see the degradation of the circular central spot to the symmetrical blurred patterns. This result shows strong agreement with the numerical results shown in Fig. 3. In addition, the astigmatism factors of Fig. 3 are proportional to the square of the tilt angles of Fig. 7. This result is in accord with the normal third-order aberration theory. These results confirm experimentally that the aberrated spot of the axicon blurred only into a cross-shape, and that the aberration of the axicon has, as a result, only even-order aberrations but no odd-order aberrations represented by coma.

5. Conclusion and discussion

We have shown that by using the PSF and the wave aberration function, the intensity

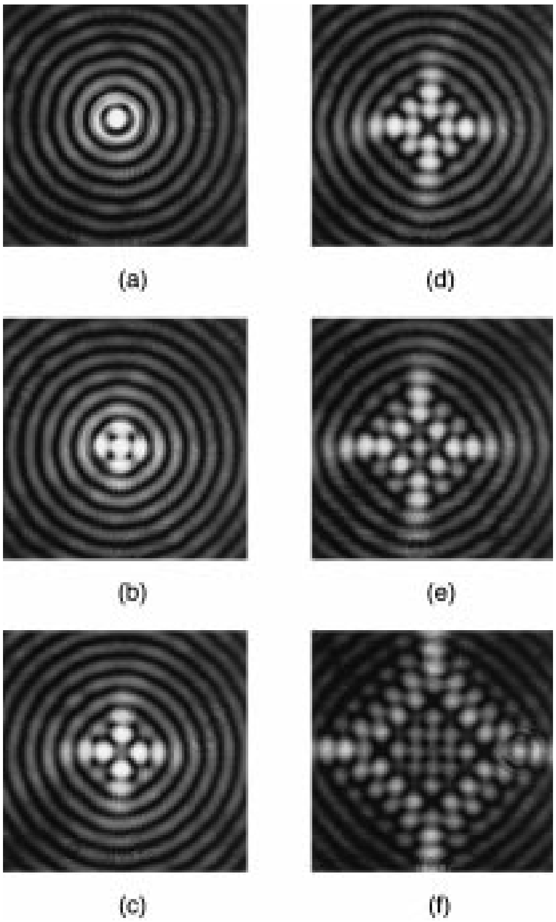

Fig. 7. Aberrated spots produced by an obliquely illuminated axicon: (a) $\theta = 0.0°$ (not aberrated), (b) $\theta = 5.0°$, (c) $\theta = 6.0°$, (d) $\theta = 7.0°$, (e) $\theta = 8.0°$, and (f) $\theta = 10.0°$.

distribution of the aberrated spot of the axicon can be derived numerically. We compared theoretically and experimentally aberrations created by an axicon with those created by a lens. We demonstrated that the intensity distributions of the aberrated light pattern created by an axicon are always symmetrical and are, therefore, characterized by an even-ordered aberration, primarily that of astigmatism. Hence, the axicon does not have an odd-ordered aberration represented by coma. The advantages of using the axicon imaging, as is well known, are its narrower central spot and long depth of focus. In addition to these properties, we confirmed that the axicon, when obliquely illuminated, produces only the astigmatism but not coma. The scanning system that employs the axicon is insensitive to coma. Therefore, for laser scanning microscopy that employs the axicon, we have to choose an astigmatism-corrected lens as relay lens, but we need not pay so much attention to the correction of the coma. This is an advantage of the axicon when employed in laser scanning microscopes, ensuring easy to designing of a lens to cover a wide scanning field as well as high resolution, long depth of focus, and high contrast imaging of the specimen.

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